Determination of Creep Moduli of Rotational Moulding grades of Polyethylene from the Expansion of Pressurised Pipe

When polyethylene is subjected to continuous stress it will stretch and will continue to do so until a point where the material will rupture. This stretching is referred to as creep and must be accounted for when designing polyethylene applications which will be subject to stress. Such applications include water tanks, where excessive creep may result in unacceptable bulging and eventual collapse.

In order to assess the rate of creep, samples of PE are typically subject to a tensile load and the rate of strain (stretching) is measured over time. The data generated enables the prediction of strain rates for longer time periods. The extrapolated strain rate for a specific time is divided into the applied stress to generate a creep modulus.

The creep modulus (E) is used by Design Engineers in the assessment of designs via Finite Element Analysis (FEA). One of the applications of FEA is the assessment of design and thickness requirements of rotationally moulded water tanks.

Creep moduli are typically calculated from data derived from tensile creep testing conducted to standards such as ASTM D2990. The samples are usually compression moulded and in our experience the data generated on PE rotational moulding grades has not been adequate in repeatability. Given the importance of generating accurate and repeatable data we decided to look for another method, one which preferably employed the production of samples by rotational moulding.

We chose to explore the possibility of generating creep data from the circumferential expansion of pressurised pipe, as this technique was already employed in the protocol for validating the Hydrostatic Design Basis (HDB) of rotational moulding grades of Polyethylene according to ASTM2837, Standard Test Method for Obtaining Hydrostatic Design Basis for Thermoplastic Pipe Materials or Pressure Design Basis for Thermoplastic Pipe Products.

There were two initial issues to be resolved when looking to utilise pressurised pipe circumferential expansion data. The first was that the circumferential expansion of a pressurised pipe is constrained to some degree by the axial stress generated by the action of pressure on the ends of the pipe. Therefore the Creep Moduli could not be simply calculated by division of the hoop stress by the hoop strain.

The second issue was that the hoop (tangential) stress is not constant across the thickness of the pipe. Thus a technique was required for determining an “average” stress across the thickness of the pipe.

Fortunately the mathematical relationships between stress and strain in closed cylinders were determined long ago and are defined by Lame’s equation. From this relationship our Dr. Steve Mirams derived an equation for the calculation of Creep Moduli. The background and derivation are detailed below.

Determination of Creep Modulus from Pipes

Theory

The following assumptions have been made:

- Thickness is uniform around the cylinder
- The material is isotropic and homogeneous
- The material behaves in a linear elastic fashion
  - This means the analysis is limited to small strains and stresses well below the yield point
- Effects of cylinder ends can be neglected
  - This limits the analysis to the middle section of cylinders with sufficient L/D ratio.
- The modulus is independent of stress
  - While this is not true for PE, this assumption will contribute only a small error over the range of conditions (i.e. pipe dimensions, internal pressures) used in the Qenos test program.

The basic relationship between stress and tangential strain for an element of a cylinder is[1]:

$$\varepsilon_t = \frac{1}{E}(\sigma_t - \nu\,\sigma_a - \nu\,\sigma_r) \qquad (1)$$

where

ε_t = tangential (hoop) strain

E = modulus

σ_t = tangential (hoop) stress

σ_a = axial stress

σ_r = radial stress

ν = Poisson's ratio.

For a sealed cylinder with internal pressure, there will be a stress in the axial direction on the ends of the cylinder. This creates strain in the axial direction that causes a contraction in the perpendicular directions through Poisson's effect. Thus, the hoop strain is reduced by the axial stress as shown in equation (1).

For thick-walled cylinders, the stresses vary across the wall. The stresses are described by Lamé's equations. In the case of a sealed cylinder where the external pressure is zero, these equations are[1]:

$$\sigma_t = \frac{a^2(r^2 + b^2)}{r^2(b^2 - a^2)} P \qquad (2)$$

$$\sigma_r = \frac{a^2(r^2 - b^2)}{r^2(b^2 - a^2)} P \qquad (3)$$

$$\sigma_a = \frac{a^2}{(b^2 - a^2)} P \qquad (4)$$

where

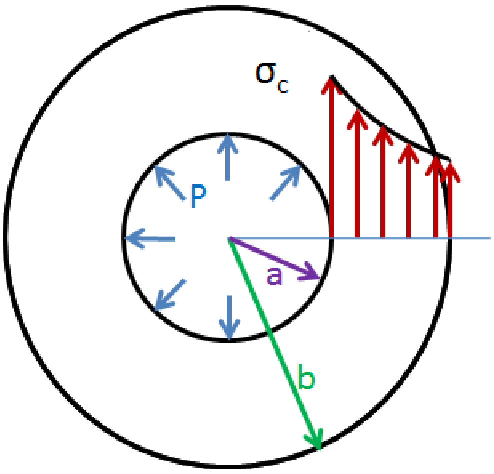
r = the radius from the axis of the cylinder

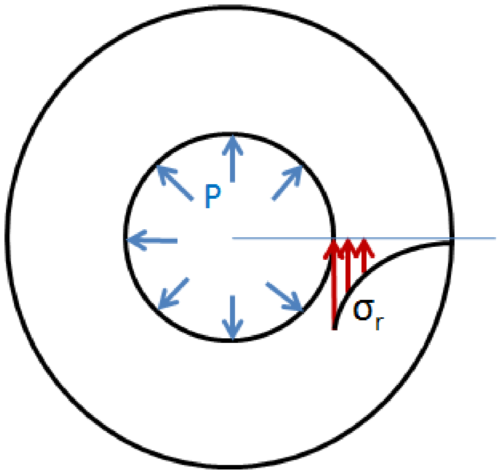
a = inside radius of cylinder

b = outside radius of cylinder

P = internal pressure

The stress variations as a function of position, r, are shown schematically in Figure 1.[2] The hoop stress is maximum at the inside radius. The radial stress is maximum in the negative direction (compressive) at the inside radius. The axial stress is constant across the wall.





At the outer circumference of the cylinder, r = b.

Substituting r = b into (2) gives the tangential stress at this outer surface, the circumferential stress, σ_c.

$$\sigma_t \text{ at outer surface } = \sigma_c = \frac{2a^2}{(b^2 - a^2)} P \qquad (5)$$

Substituting r = b into (3) gives the radial stress at the outer surface: $\sigma_r = 0$

Substituting (4) and (5) into (1) gives the circumferential strain, ε_c

$$\varepsilon_t \text{ at outer surface } = \varepsilon_c = \frac{1}{E}\frac{a^2(2-\nu)}{(b^2 - a^2)} P \qquad (6)$$

Rearranging, the modulus can be calculated from:

$$E = \frac{a^2(2-\nu)}{\varepsilon_c(b^2 - a^2)} P \qquad (7)$$

Cross-checks

First cross-check

As a cross-check, these equations can be compared to those used for thin-walled cylinders. The approximation used for hoop stress in thin-walled cylinders is

$$\sigma_t = \frac{(OD - t)}{2t} P \qquad (8)$$

Where,

P = internal pressure,

OD = outer diameter of cylinder (= 2b),

t = wall thickness of cylinder (= b – a)

The approximation for the modulus in a thin walled cylinder is[1]

$$E = \frac{a^2(2-\nu)}{\varepsilon_c(b^2 - a^2)} P \qquad (7)$$

It can be shown that equation (8) approaches equation (5) as the wall thickness becomes smaller (i.e. as b approaches a).

It can also be shown that using equation (8) will result in higher values of E than equation (5). Hence using equation (7) will provide more conservative values of the modulus than the standard approximations for thin-walled cylinders.

Second cross-check

An alternative expression for the modulus as a function of the radius has been published[3]:

$$r\varepsilon_t = \frac{(1+\nu)a^2 P}{E(b^2 - a^2)}\left[\frac{(1-2\nu)r}{(1+\nu)} + \frac{b^2}{r}\right] \qquad (9)$$

Rearranging this and inserting the condition at the circumference, r = b, this becomes:

$$E = \frac{(1+\nu)a^2 P}{\varepsilon_c b(b^2 - a^2)}\left[\frac{(1-2\nu)b}{(1+\nu)} + b\right] \quad (10)$$

Multiplying out and collecting terms, this equation reduces to equation (7).

Thus we now have an equation (denoted as Mirams' Equation) which enables the calculation of the creep modulus (E) at any point in time and is stated as follows.

$$E = \frac{a^2(2-\nu)}{\varepsilon_c(b^2 - a^2)} P$$

Where P= internal pressure

ε_c = circumferential strain

a = internal pipe radius

b = external pipe radius.

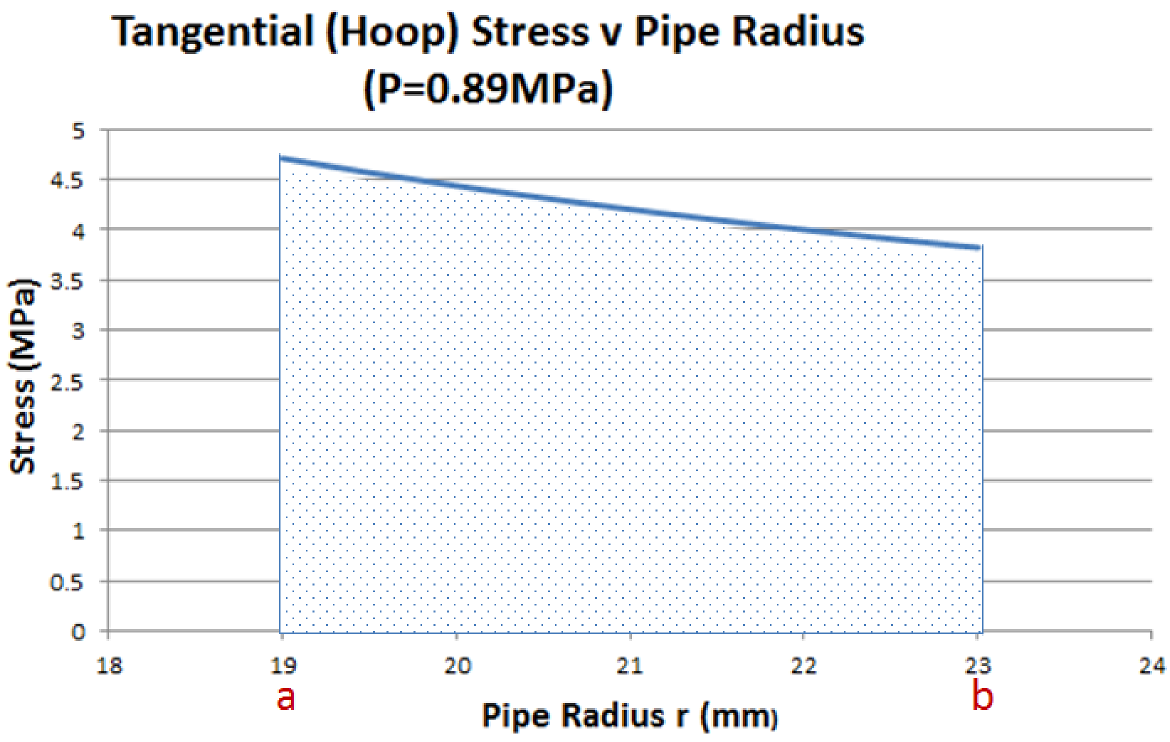
ν = Poisson's ratio

Derivation of an equation for the calculation of "average" tangential (hoop) stress

The second issue, that of determining an "average" stress across the thickness of the pipe was resolved by the integration of Lame's equation for the determination of hoop (tangential) stress of thick walled cylinders.

tangential (hoop) stress $\sigma_t = \dfrac{a^2(r^2 + b^2)}{r^2(b^2 - a^2)} P$

The following diagram represents a typical set of variables and the resulting stress curve.



The diagram above shows an example of the stress levels generated in a pipe with internal radius of 19.0 mm and an external radius of 23.2mm. Dividing the area under the curve by the pipe thickness provides the average tangential (hoop) stress.

The area under the curve may be determined by integrating Lame's equation between the inner radius (a) and the outer radius (b).

Frank Austin of Canada College, San Francisco was kind enough to determine the following integral.

$$\int_a^b \frac{a^2(r^2 + b^2)}{r^2(b^2 - a^2)} P \, dr \; = aP$$

The area under the curve is therefore equal to the inner radius multiplied by the pressure.

Therefore the average tangential (hoop) stress may be calculated as follows.

$\sigma_{t(ave)}$ = aP/(b-a)

The pressure required to generate a specific average stress may then be calculated using the following equation.

$P = \sigma_{t(ave)}.(b-a)/a$

Verification of Mirams' Equation

In order to verify Mirams' Equation we employed the services of DB Consulting, a major engineering consultant to the Rotational Moulding industry. We set David Beneke the task of determining the apparent modulus of Alkatuff LL711UV via Finite Element Analysis modelling, given two sets of actual data derived from pressurised pipe testing conducted in the Qenos Technical Centre.

The two sets of data were chosen to cover a range of stresses and times to ensure the equation was valid as both were varied.

The following table has been extracted from the resultant report from DB Engineering. The modulus results are highlighted.

Test No.	Time (Hours)	Internal Pressure (MPa)	Initial OD (mm)	Final OD (mm)	Target Delta R (mm)	FEA Apparent Modulus (MPa)	FEA $\sigma_{circ.avg}$ (MPa)
1	672	0.867	46.35	47.59	0.620	**108.9**	4.05
2	1488	1.297		48.84	1.245	**81.1**	6.06

The value for Poisson's ratio used in this modelling was 0.39.

When the same data was applied to Miram's Equation the following results were achieved.

Case 1

E =19.055^2*(2-0.39)/((47.59-46.35)/46.35*(23.175^2-19.055^2))*0.867 = 108.9 MPa

Case 2

E =19.055^2*(2-0.39)/((48.84-46.35)/46.35*(23.175^2-19.055^2))*1.297 = 81.1 MPa

The results were identical to those generated via FEA modelling. Mirams' Equation was therefore verified.

With the method now verified, Qenos have undertaken work to generate Creep Data and Moduli for Alkatuff LL711UV. The information derived from initial work conducted at 40°C is now available for design engineers.

References:

1. LS Negi, *Strength of Materials*, Tata McGraw-Hill Education, 2008, pp. 255-257.
2. University of Washington, *Pressure Vessels: Combined Stresses,* 2007, p 12.6, viewed at: http://courses.washington.edu/me354a/chap12.pdf
3. WF Chen & DJ Han, *Plasticity for Structural Engineers*, J Ross Publishing, 2007, p. 199.